

SPM BULLETIN

ISSUE NUMBER 23: December 2007 CE

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1. EDITOR'S NOTE

A surprising number of new results and directions in “core” SPM by Babinkostova and Scheepers in the last quarter of the year! See §§2.2–2.3 and 2.11–2.13.

Have a good 2008,

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2. RESEARCH ANNOUNCEMENTS

2.1. Cardinal invariants of the continuum and combinatorics on uncountable cardinals. We explore the connection between combinatorial principles on uncountable cardinals, like stick and club, on the one hand, and the combinatorics of sets of reals and, in particular, cardinal invariants of the continuum, on the other hand. For example, we prove that additivity of measure implies that Martins axiom holds for any Cohen algebra. We construct a model in which club holds, yet the covering number of the null ideal $\text{cov}(\mathcal{N})$ is large. We show that for uncountable cardinals $\kappa \leq \lambda$ and $\mathcal{F} \subseteq [\lambda]^\kappa$, if all subsets of λ either contain, or are disjoint from, a member of \mathcal{F} , then \mathcal{F} has size at least $\text{cov}(\mathcal{N})$ etc. As an application, we solve the Gross space problem under $c = \aleph_2$ by showing that there is such a space over any countable field. In two appendices, we solve problems of Fuchino, Shelah and Soukup, and of Kraszewski, respectively.

Jörg Brendle

2.2. Selection principles and countable dimension. We characterize countable dimensionality and strong countable dimensionality by means of an infinite game.

<http://arxiv.org/abs/0709.2893>

Liljana Babinkostova and Marion Scheepers

2.3. Products and selection principles. The product of a Sierpinski set and a Lusin set has Menger's property. The product of a gamma set and a Lusin set has Rothberger's property.

<http://arxiv.org/abs/0709.2895>

Liljana Babinkostova and Marion Scheepers

2.4. On completely donut (doughnut) sets. A set $\langle A, B \rangle = \{X \in [\omega]^\omega : A \subseteq X \subseteq B\}$ is a donut, whenever $A \subseteq B \subseteq \omega$ and $B \setminus A$ is infinite. A subset $S \subseteq [\omega]^\omega$ is completely donut, whenever for each donut $\langle A, B \rangle$ there exists a donut $\langle C, D \rangle \subseteq \langle A, B \rangle$ such that $\langle C, D \rangle \subseteq S$ or $\langle C, D \rangle \cap S = \emptyset$. If always holds $\langle C, D \rangle \cap S = \emptyset$, then S is nowhere donut. We examine families of completely donut and nowhere donut sets. The results correspond to completely Ramsey and nowhere Ramsey sets.

<http://arxiv.org/abs/0709.3016>

Piotr Kalembe, Szymon Plewik and Anna Wojciechowska

2.5. On minimal non-potentially closed subsets of the plane. We study the Borel subsets of the plane that can be made closed by refining the Polish topology on the real line. These sets are called potentially closed. We first compare Borel subsets of the plane using products of continuous functions. We show the existence of a perfect antichain made of minimal sets among non-potentially closed sets. We apply this result to graphs, quasi-orders and partial orders. We also give a non-potentially closed set minimum for another notion of comparison. Finally, we show that we cannot have injectivity in the Kechris-Solecki-Todorćević dichotomy about analytic graphs.

Topology and its Applications 154 (2007), 241–262.

<http://arxiv.org/abs/0710.0152>

*Dominique Lecomte*¹

2.6. One Dimensional Locally Connected S -spaces. We construct, assuming Jensen’s principle \diamond , a one-dimensional locally connected hereditarily separable continuum without convergent sequences. The construction is an inverse limit in ω_1 steps, and is patterned after the original Fedorchuk construction of a compact S -space. To make it one-dimensional, each space in the inverse limit is a copy of the Menger sponge.

<http://arxiv.org/abs/0710.1085>

Joan E. Hart Kenneth Kunen

2.7. Covering an uncountable square by countably many continuous functions. We prove that there exists a countable family of continuous real functions whose graphs together with their inverses cover an uncountable square, i.e. a set of the form $X \times X$, where X is uncountable. This is motivated by an old result of Sierpiński, saying that $\aleph_1 \times \aleph_1$ is covered by countably many graphs of functions and inverses of functions. Another motivation comes from Shelah’s study of planar Borel sets without perfect rectangles.

<http://arxiv.org/abs/0710.1402>

Wiesław Kubis

2.8. Ramsey degrees of finite ultrametric spaces, ultrametric Urysohn spaces and dynamics of their isometry groups. We study Ramsey-theoretic properties of several natural classes of finite ultrametric spaces, describe the corresponding Urysohn spaces and compute a dynamical invariant attached to their isometry groups.

<http://arxiv.org/abs/0710.2347>

L. Nguyen Van Thé

¹Lecomte has recently uploaded quite a few interesting papers to the ArXiv. Check there.

2.9. Big Ramsey degrees and divisibility in classes of ultrametric spaces.

Given a countable set S of positive reals, we study finite-dimensional Ramsey-theoretic properties of the countable ultrametric Urysohn space with distances in S .

<http://arxiv.org/abs/0710.2352>

L. Nguyen Van Thé

2.10. Definable Davies' Theorem. We prove the following analogue of a Theorem of R.O. Davies: Every Σ_2^1 function $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ can be represented as a sum of rectangular Σ_2^1 functions if and only if all reals are constructible.

<http://arxiv.org/abs/0711.0162>

Asger Tornquist and William Weiss

2.11. Selection Principles and Baire spaces. We prove that if X is a separable metric space with the Hurewicz covering property, then the Banach-Mazur game played on X is determined. The implication is not true when “Hurewicz covering property” is replaced with “Menger covering property”.

<http://arxiv.org/abs/0711.1104>

Marion Scheepers

2.12. Selective screenability in topological groups. We examine the selective screenability property in topological groups. In the metrizable case we also give characterizations in terms of the Haver property and finitary Haver property respectively relative to left-invariant metrics. We prove theorems stating conditions under which the properties are preserved by products. Among metrizable groups we characterize the ones of countable covering dimension by a natural game.

<http://arxiv.org/abs/0711.1322>

Liljana Babinkostova

2.13. Selective screenability and the Hurewicz property. We characterize the Hurewicz covering property in metrizable spaces in terms of properties of the metrics of the space. Then we show that a weak version of selective screenability, when combined with the Hurewicz property, implies selective screenability.

<http://arxiv.org/abs/0711.1516>

Liljana Babinkostova

2.14. Partitioning triples and partially ordered sets.

<http://www.ams.org/journal-getitem?pii=S0002-9939-07-09170-8>

Albin Jones

2.15. A polarized partition relation for cardinals of countable cofinality.

<http://www.ams.org/journal-getitem?pii=S0002-9939-07-09143-5>

Albin Jones

2.16. On topological spaces of singular density and minimal weight. In a recent paper, Juhász and Shelah establish the consistency of a regular hereditarily Lindelöf space of density \aleph_{ω_1} .

It is natural to ask what is the minimal possible value for the weight of such space; more specifically, can the weight be \aleph_{ω_1} ?

In this paper, we isolate a certain consequence of the Generalized Continuum Hypothesis, which we will refer to as the *Prevalent Singular Cardinals Hypothesis*, and show it implies that every topological space of density and weight \aleph_{ω_1} is not hereditarily Lindelöf.

The assertion PSH is very weak, and in fact holds in all currently known models of ZFC.

dx.doi.org/10.1016/j.topol.2007.09.013

Assaf Rino

2.17. There is a van Douwen MAD family. We prove in ZFC that there is a MAD family of functions in $\mathbb{N}^{\mathbb{N}}$ which is also maximal with respect to infinite partial functions. This solves a long standing question of van Douwen. We also prove that such families cannot be analytic. This strengthens Steprans' result that strongly MAD families cannot be analytic.

<http://arxiv.org/abs/0711.4400>

Dilip Raghavan

2.18. Cardinal sequences of LCS spaces under GCH. We give full characterization of the sequences of regular cardinals that may arise as cardinal sequences of locally compact scattered spaces under GCH. The proofs are based on constructions of universal locally compact scattered spaces.

<http://arxiv.org/abs/0712.0584>

Juan Carlos Martinez, Lajos Soukup

2.19. Dirichlet sets and Erdos-Kunen-Mauldin theorem. By a theorem proved by Erdos, Kunen and Mauldin, for any nonempty perfect set P on the real line there exists a perfect set M of Lebesgue measure zero such that $P + M = \mathbb{R}$. We prove a stronger version of this theorem in which the obtained perfect set M is a Dirichlet set. Using this result we show that for a wide range of families of subsets of the reals, all additive sets are perfectly meager in transitive sense. We also prove that every proper analytic subgroup G of the reals is contained in an F_σ set F such that $F + G$ is a meager null set.

<http://arxiv.org/abs/0712.2112>

Peter Elias

2.20. Local Ramsey theory: An abstract approach. It is shown that the known notion of selective coideal can be extended to a family \mathcal{H} of subsets of \mathcal{R} , where (\mathcal{R}, \leq, r) is a topological Ramsey space in the sense of Todorcevic. Then it is proven

that, if \mathcal{H} selective, the \mathcal{H} -Ramsey and \mathcal{H} -Baire subsets of \mathcal{R} are equivalent. This extends results of Farah for semiselective coideals of \mathbb{N} . Also, it is proven that the family of \mathcal{H} -Ramsey subsets of \mathcal{R} is closed under the Souslin operation.

<http://arxiv.org/abs/0712.2393>

José Mijares and Jesús Nieto

2.21. There are no hereditary productive γ -spaces. We show that if X is an uncountable productive γ -set [F. Jordan, Productive local properties of function spaces, *Topology Appl.* **154**, 870–883, 2007], then there is a countable $Y \subseteq X$ such that $X \setminus Y$ is not Hurewicz.

Along the way we will prove a general result about Fréchet- α_2 filters to gain information about countable unions of γ -spaces and productive γ -spaces. In particular, we answer a question of A. Miller by showing that an increasing countable union of γ -spaces is again a γ -space. We also use recent methods of B. Tsaban and L. Zdomskyy to show that λ -spaces with Hurewicz property are precisely those spaces for which every co-countable set is Hurewicz.

Francis Jordan

3. PROBLEM OF THE ISSUE

For a sequence $\{X_n\}_{n \in \mathbb{N}}$ of subsets of X , define $\liminf X_n = \bigcup_m \bigcap_{n \geq m} X_n$. For a family \mathcal{F} of subsets of X , $L(\mathcal{F})$ denotes its closure under the operation \liminf . X has the δ -property if for each open ω -cover \mathcal{U} of X , $X \in L(\mathcal{U})$. This property was introduced by Gerlits and Nagy in their seminal paper [1].

Clearly, the γ -property $\left(\frac{\Omega}{\Gamma}\right)$ implies the δ -property. $S_1(\Omega, \Gamma) = \left(\frac{\Omega}{\Gamma}\right)$ [1].

Problem 3.1 (Gerlits-Nagy [1]). *Is the δ -property equivalent to $\left(\frac{\Omega}{\Gamma}\right)$?*

Miller suggested that, as a union of an increasing sequence of sets with the δ -property has again the δ -property, one can obtain a negative answer by finding an increasing sequence $\{X_n\}_{n \in \mathbb{N}}$ of sets, each satisfying $\left(\frac{\Omega}{\Gamma}\right)$, such that their union does not satisfy $\left(\frac{\Omega}{\Gamma}\right)$. Recently, Francis Jordan solved Miller's question by proving that this is impossible (Section 2.21 above).

A closely-related problem, due to Sakai, concerns the Pytkeev property in function spaces. This problem asks whether, for each set of reals X , if $C_p(X)$ has the Pytkeev property then $C_p(X)$ is Fréchet (i.e., X satisfies $\left(\frac{\Omega}{\Gamma}\right)$) – see [2] for details.

Boaz Tsaban

REFERENCES

- [1] J. Gerlits and Zs. Nagy, *Some properties of $C(X)$, I*, *Topology and its Applications* **14** (1982), 151–161.
- [2] M. Sakai, *Special subsets of reals characterizing local properties of function spaces*, in: **Selection Principles and Covering Properties in Topology** (Lj. D.R. Kočinac, ed.), *Quaderni di Matematica* **18** (2007), 195–225.

4. UNSOLVED PROBLEMS FROM EARLIER ISSUES

- Issue 1.** Is $\left(\frac{\Omega}{\Gamma}\right) = \left(\frac{\Omega}{\Gamma}\right)$?
- Issue 2.** Is $\mathcal{U}_{fin}(\mathcal{O}, \Omega) = \mathcal{S}_{fin}(\Gamma, \Omega)$? And if not, does $\mathcal{U}_{fin}(\mathcal{O}, \Gamma)$ imply $\mathcal{S}_{fin}(\Gamma, \Omega)$?
- Issue 4.** Does $\mathcal{S}_1(\Omega, \Gamma)$ imply $\mathcal{U}_{fin}(\Gamma, \Gamma)$?
- Issue 5.** Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)
- Issue 6.** Does there exist (in ZFC) an uncountable set satisfying $\mathcal{S}_{fin}(\mathcal{B}, \mathcal{B})$?
- Issue 8.** Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \mathcal{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?
- Issue 9 (CH).** Is $\text{Split}(\Lambda, \Lambda)$ preserved under finite unions?
- Issue 10.** Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)
- Issue 11.** Does $\mathcal{S}_1(\Gamma, \Gamma)$ always contain an element of cardinality \mathfrak{b} ?
- Issue 12.** Could there be a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into \aleph_1 meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M ?
- Issue 14.** Does there exist (in ZFC) a set of reals X of cardinality \mathfrak{d} such that all finite powers of X have Menger's property $\mathcal{S}_{fin}(\mathcal{O}, \mathcal{O})$?
- Issue 15.** Can a Borel non- σ -compact group be generated by a Hurewicz subspace?
- Issue 16 (MA).** Is there an uncountable $X \subseteq \mathbb{R}$ satisfying $\mathcal{S}_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?
- Issue 17 (CH).** Is there a totally imperfect X satisfying $\mathcal{U}_{fin}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^\mathbb{N}$?
- Issue 18 (CH).** Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?
- Issue 19.** Does the Pytkeev property of $C_p(X)$ imply that X has Menger's property?
- Issue 20.** Does every hereditarily Hurewicz space satisfy $\mathcal{S}_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$?
- Issue 21 (CH).** Is there a Rothberger-bounded $G \leq \mathbb{Z}^\mathbb{N}$ such that G^2 is not Menger-bounded?
- Issue 22.** Let \mathcal{W} be the van der Waerden ideal. Are \mathcal{W} -ultrafilters closed under products?
- Issue 23.** Is the δ -property equivalent to the γ -property $\left(\frac{\Omega}{\Gamma}\right)$?

Previous issues. The previous issues of this bulletin are available online at <http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22>

Contributions. Announcements, discussions, and open problems should be emailed to tsaban@math.biu.ac.il

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